

FOR OFFICIAL USE

P.1

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. If we choose a number randomly from the factors of $13!$, the probability that it is an odd number is $\frac{1}{A}$. Find the value of A .

如果我们从 $13!$ 的因子中随机选择一个数，这个数是奇数的概率是 $\frac{1}{A}$ ，求 A 的值。

$A =$

2. If a and b are two different positive integers such that $ab = a + b + A + 1$, find the value of $B = a + b$.

如果 a 和 b 是两个不同的正整数，使得 $ab = a + b + A + 1$ ，求 $B = a + b$ 的值。

$B =$

3. Let C be the number of positive integers less than or equal to $31B$ that are divisible by neither 3 nor 5. Find C .

设 C 为小于或等于 $31B$ 且既不能被 3 也不能被 5 整除的正整数的数量，求 C 。

$C =$

4. Let $D = \frac{C}{53}$. Solve for the integer x such that $\log_D x + \log_D (x - 20) = 3$.

设 $D = \frac{C}{53}$ 。求整数解 x 使 $\log_D x + \log_D (x - 20) = 3$ 。

$D =$

Hong Kong Mathematics Olympiad (2024/25)
Finals (Individual – Event 2) (Modified by EDB)

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			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
								<input type="text"/>
							Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. Let A be the set of all positive integers n such that $n^2 + 2n$ is a perfect square and α be the number of elements of A . Find α .

设 A 为所有正整数 n 的集合，且 $n^2 + 2n$ 为完全平方数及 α 为 A 中元素个数。求 α 。

$\alpha =$

2. Let G be the set of positive integers which are smaller than 1000, where $\alpha + 3$, $\alpha + 5$ and $\alpha + 7$ are not factors. Let b be the number of elements in G and $\beta = \left\lfloor \frac{b}{100} \right\rfloor$, where $[x]$ denotes the largest integer not exceeding x . Find β .

设 G 为小于 1000 的正整数集合，其中 $\alpha + 3$ 、 $\alpha + 5$ 和 $\alpha + 7$ 不是因子。设 b 为 G 的元素个数及 $\beta = \left\lfloor \frac{b}{100} \right\rfloor$ ，其中 $[x]$ 为不超过 x 的最大整数。求 β 。

$\beta =$

3. Given that x , y and z satisfy

$$x - y + z = 1$$

$$\frac{x}{2\beta} - \frac{y}{\beta} + \frac{z}{2} = 1$$

$$\frac{x}{27} - \frac{y}{9} + \frac{z}{3} = 1$$

Let $\gamma = \left\lfloor \frac{xyz}{100} \right\rfloor$, where $[x]$ denotes the largest integer not exceeding x . Find γ .

已知 x 、 y 和 z 满足

$$x - y + z = 1$$

$$\frac{x}{2\beta} - \frac{y}{\beta} + \frac{z}{2} = 1$$

$$\frac{x}{27} - \frac{y}{9} + \frac{z}{3} = 1$$

设 $\gamma = \left\lfloor \frac{xyz}{100} \right\rfloor$ ，其中 $[x]$ 为不超过 x 的最大整数。求 γ 。

$\gamma =$

4. Let $\triangle ABC$ be a triangle with sides $a = BC$, $b = AC$ and $c = AB$. The in-circle of $\triangle ABC$ is tangent to BC , AC and AB at D , E and F respectively. If $BD = \gamma$, $DC = \gamma + 1$ and $AE = \gamma + 2$, find the perimeter of $\triangle ABC$.

设 $\triangle ABC$ 为一个三角形，其边为 $a = BC$ 、 $b = AC$ 、 $c = AB$ 。 $\triangle ABC$ 的内切圆分别与 BC 、 AC 及 AB 相切于点 D 、 E 及 F 。 若 $BD = \gamma$ 、 $DC = \gamma + 1$ 且 $AE = \gamma + 2$ ，求 $\triangle ABC$ 的周长。

$D =$

Hong Kong Mathematics Olympiad (2024/25)
Finals (Individual – Event 3) (Modified by EDB)

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			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
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							Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. Given that $x^2 + y^2 - 27x + 54y = 2025$ and $A_1 = x + y$, find the value of A_1 .

已知 $x^2 + y^2 - 27x + 54y = 2025$ 及 $A_1 = x + y$ ，求 A_1 的值。

$A_1 =$

2. Given that $\sin \theta (\sin \theta + \cos \theta) = \frac{\sqrt{A_1} - 1}{\sqrt{A_1} + 2}$ and $A_2 = \tan \theta$, where $0^\circ \leq \theta \leq 90^\circ$. Find the value of A_2 .

已知 $\sin \theta (\sin \theta + \cos \theta) = \frac{\sqrt{A_1} - 1}{\sqrt{A_1} + 2}$ 及 $A_2 = \tan \theta$ ，其中 $0^\circ \leq \theta \leq 90^\circ$ ，求 A_2 的值。

$A_2 =$

3. Given that x and y are positive integers which satisfy $\frac{20}{x} - \frac{2}{y} = \frac{1}{A_2}$. If A_3 is the sum of all possible values of x , find the value of A_3 .

已知 x 及 y 为满足方程 $\frac{20}{x} - \frac{2}{y} = \frac{1}{A_2}$ 的正整数。若 A_3 是所有 x 的可能值之和，求 A_3 的值。

$A_3 =$

4. In a triangle ABC , $\angle BAC = 45^\circ$, $\angle ABC = 75^\circ$, $AC = \sqrt{A_3 + 1}$. If A_4 is the area of the triangle, find A_4 .

在三角形 ABC 中， $\angle BAC = 45^\circ$ ， $\angle ABC = 75^\circ$ ， $AC = \sqrt{A_3 + 1}$ 。若 A_4 是该三角形的面积，求 A_4 。

$A_4 =$

Hong Kong Mathematics Olympiad (2024/25)
Finals (Individual – Event 4) (Modified by EDB)

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除非特别声明，答案须用数字表达，并化至最简。

1. Let a be a positive real number such that $A = a + \frac{1}{a}$. If $a^3 + \frac{1}{a^3} = 110$, find the value of A .

设 a 为正实数，且 $A = a + \frac{1}{a}$ 。若 $a^3 + \frac{1}{a^3} = 110$ ，求 A 的值。

$A =$

2. Given that $B = \sqrt{1 + (2020 + A)^2 + \frac{2025^2}{2026^2}} - \frac{1}{2026}$, find the value of B .

已知 $B = \sqrt{1 + (2020 + A)^2 + \frac{2025^2}{2026^2}} - \frac{1}{2026}$ ，求 B 的值。

$B =$

3. Given that C and y are positive integers and $C < y$. If $Cy - C - y = B$, find the value of C .

已知 C 及 y 是正整数且 $C < y$ 。若 $Cy - C - y = B$ ，求 C 的值。

$C =$

4. If $f(x) = \frac{9^x}{9^x + C}$, find $f\left(\frac{1}{2025}\right) + f\left(\frac{2}{2025}\right) + f\left(\frac{3}{2025}\right) + \cdots + f\left(\frac{2024}{2025}\right)$.

设 $f(x) = \frac{9^x}{9^x + C}$ ，求 $f\left(\frac{1}{2025}\right) + f\left(\frac{2}{2025}\right) + f\left(\frac{3}{2025}\right) + \cdots + f\left(\frac{2024}{2025}\right)$ 。

$D =$